Cosmological constraints on modified gravity theories

Diego Sáez-Gómez

Astrophysics, Cosmology and Gravity Center
University of Cape Town

and

Department of Theoretical Physics
University of the Basque Country

August 3, 2013

Kobayashi-Maskawa Institute
University of Nagoya

Index

- Friedmann-Lemaître-Robertson-Walker (FLRW) cosmologies in F(R) gravity
- The cosmological evolution in viable F(R) gravities, and their scalar-tensor counterpart. Future singularities and Little Rip.
- Testing F(R) gravity with SNe Ia data: a simple model
- F(R,T) gravity and cosmological perturbations

FLRW COSMOLOGIES

- **Cosmological Principle**: assumes that the Universe is the same in every point (at large scales), what from a mathematical point of view, it means that the Universe is homogeneous and isotropic.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - k\ r^2} + d\Omega^2\right]$$

- **An expanding Universe that starts in a initial singularity.**

- **Hot initial state: Primordial Nucleosyntesis.**

- **Last scattering surface: Cosmic Microwave Background (CMB).**

- **Formation of Large Scale Structure.**

FLRW cosmologies

General Relativity: Big Bang model

FLRW equations in General Relativity

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}\,,$$
$$\dot{H} = -4\pi G(p+\rho) + \frac{K}{a^2}\,,$$

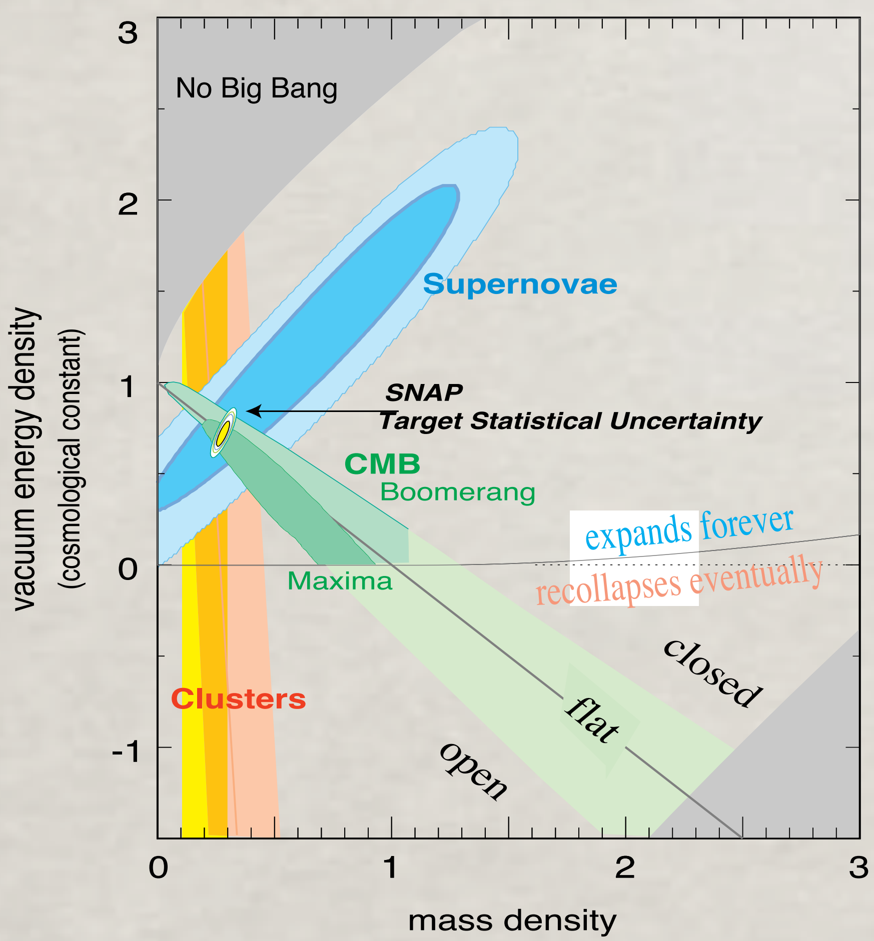
For a perfect fluid with an equation of state, $w = p/\rho$

$$H = \frac{2}{3(1+w)(t-t_0)}\,,$$
$$a(t) \propto (t-t_0)^{\frac{2}{3(1+w)}}\,,$$
$$\rho \propto a^{-3(1+w)}\,,$$

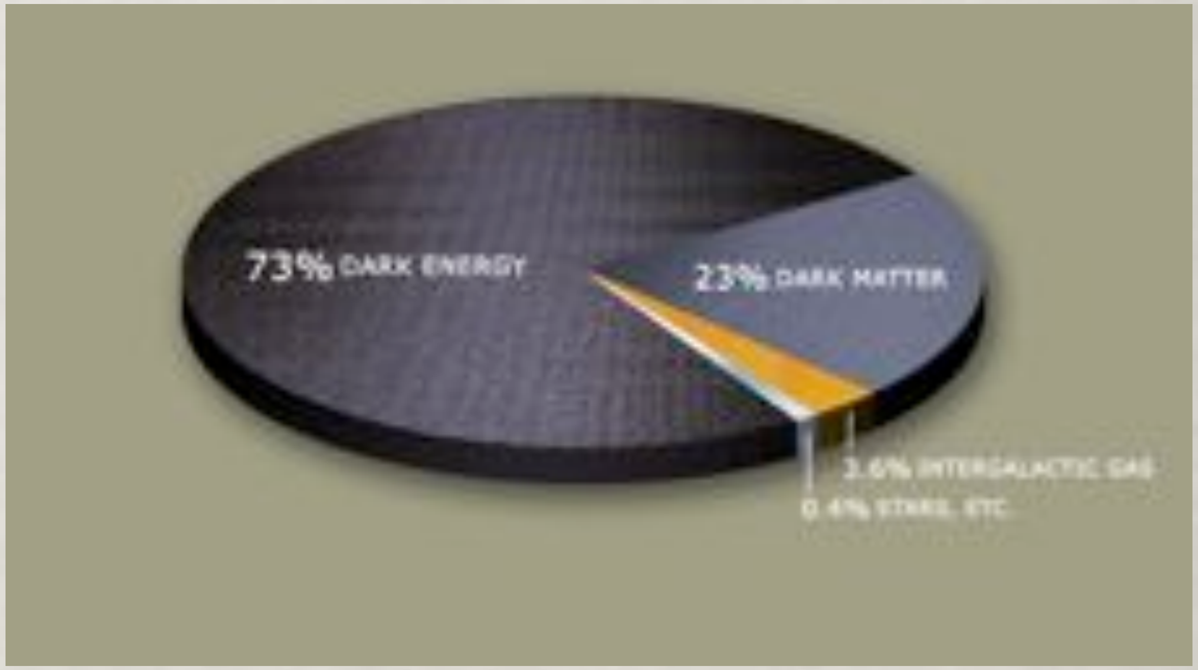
which contains an initial singularity (and perhaps a future one..).

$$\text{Radiation}: \quad a(t) \propto (t-t_0)^{1/2}\,, \quad \rho \propto a^{-4}\,,$$
$$\text{Dust}: \quad a(t) \propto (t-t_0)^{2/3}\,, \quad \rho \propto a^{-3}\,.$$

FLRW COSMOLOGIES



G. Aldering [SNAP Collaboration], "Future Research Direction and Visions for Astronomy", Alan M. Dressler, editor, Proceedings of the SPIE, Volume 4835, pp. 146-157 [arXiv:astro-ph/0209550].



We need something else: **Dark energy or modified gravity....but how is the Equation of State?**

FLRW COSMOLOGIES

Dark energy equation of state $$p = w\rho$$

$$\begin{aligned} w &> -1, \quad \text{quintessence fluid} \\ w &= -1, \quad \text{cosmological constant} \\ w &< -1, \quad \text{phantom fluid} \end{aligned}$$

Einstein gravity

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\ \sqrt{-g}\ R$$

$$+$$

Dark energy

$$S = -\int d^4x\ \sqrt{-g}\ \Lambda$$

$$S = \int d^4x\ \sqrt{-g}\ \left(\pm\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

$$S = -\frac{1}{4}\int d^4x\ \sqrt{-g}\ F_{\mu\nu}F^{\mu\nu}$$

$$S = \int d^4x\sqrt{-g}\ F(R)$$

$$S = \int d^4x\sqrt{-g}F(R,G)$$

$$S = \int d^4x\sqrt{-g}\ F(R,T)$$

$$S = \int d^4x\sqrt{-g}F(R,T,R_{\mu\nu}T^{\mu\nu})$$

E. J. Copeland, M. Sami, and S. Tsujikawa. Dynamics of Dark Energy. Int. J. Mod. Phys. D, 15:1753–1935, 2006, arXiv:hep-th/0603057S. Nojiri and S. D. Odintsov, eConf C0602061, 06 (2006); hep-th/0601213; arXiv: 0807.0685; S.~Capozziello and M. De Laurentis, Phys. Rept. 509, 167 (2011), arXiv:1108.6266; S. Capozziello and V. Faraoni, Beyond Einstein Gravity, Fundamental Theories of Physics Vol. 170, Springer Ed., Dordrecht (2011); V. Sahni and A. Starobinsky. Reconstructing Dark Energy. Int. J. Mod. Phys. D, 15:2105–2132, 2006, arXiv:astro-ph/0610026 A.de la Cruz-Dombriz and DSG, Entropy {\bf 14}, 1717 (2012) [arXiv:1207.2663 [gr-qc]], S. Odintsov, D. Sáez-Gómez, arXiv:1304.5411....

FLRW cosmologies

Future singularities

- Type I ("Big Rip"): For $t \to t_s$, $a \to \infty$ and $\rho \to \infty$, $|p| \to \infty$.
- Type II ("Sudden"): For $t \to t_s$, $a \to a_s$ and $\rho \to \rho_s$, $|p| \to \infty$.
- Type III: For $t \to t_s$, $a \to a_s$ and $\rho \to \infty$, $|p| \to \infty$.
- Type IV: For $t \to t_s$, $a \to a_s$ and $\rho \to \rho_s$, $p \to p_s$ but higher derivatives of Hubble parameter diverge.

J. D. Barrow, G. J. Galloway and F. J. Tipler, Mon. Not. Roy. Astr. Soc. 223, 835-844 (1986); R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91 071301 (2003), astro-ph/0302506; S. Nojiri, S. Odintsov, S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005); arXiv:hep-th/0501025; M. P. Dabrowski and T. Denkieiwcz, Phys. Rev. D 79, 063521 (2009) [arXiv:0902.3107 [gr-qc]]....

F(R) GRAVITY

Action,

$$S_{EH} = \int d^4x\sqrt{-g}\left[R + 2\kappa^2\mathcal{L}_m\right] \quad \rightarrow \quad S = \int d^4x\sqrt{-g}\left[f(R) + 2\kappa^2\mathcal{L}_m\right]$$

Field equations,

$$R_{\mu\nu}f_R(R) - \frac{1}{2}g_{\mu\nu}f(R) + g_{\mu\nu}\Box f_R(R) - \nabla_\mu\nabla_\nu f_R(R) = \kappa^2 T^{(m)}_{\mu\nu}$$

where as usual the energy-momentum tensor is defined as: $T^{(m)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}$

Spatially flat FLRW equations,

$$H^2 = \frac{1}{3f_R}\left[\kappa^2\rho_m + \frac{Rf_R - f}{2} - 3H\dot{R}f_{RR}\right]$$

$$-3H^2 - 2\dot{H} = \frac{1}{f_R}\left[\kappa^2 p_m + \dot{R}^2 f_{RRR} + 2H\dot{R}f_{RR} + \ddot{R}f_{RR} + \frac{1}{2}(f - Rf_R)\right]$$

Viable f(R) gravity

Viability conditions for *F(R)* gravity

1. $f_{RR} > 0$ for hight curvature, which ensures the existence of stable high curvature regimes, as the matter dominated epoch.

2. $1 + f_R > 0$, which avoids the appearance of a negative effective gravitational coupling $G_{eff} = G/(1 + f_R)$, and the anti-gravity regime, implying also the avoidance of the graviton to turn into a ghost.

3. $f_R \to 0$ as $R \to \infty$. GR has to be recovered at early times. Together with $f_{RR} > 0$, implies $f_R < 0$, and consequently $-1 < f_R < 0$.

4. $|f_R| \ll 1$ at recent epochs, which is imposed by local gravity tests. Nevertheless, it is not clear the limit for this constraint.

5. The action should have a very particular form in order to avoid the Dolgov-Kawasaki instability.

L. Pogosian and A. Silvestri, Phys. Rev. D77 023503, (2008).
A. D. Dolgov and M. Kawasaki, Phys. Lett. B 573, 1 (2003).

Viable f(R) gravity

Some viable *f(R)* gravity models

$$f(R) = R - R_{HS}\frac{c_1(R/R_{HS})^n}{c_2(R/R_{HS})^n + 1}$$

W. Hu and I. Sawicki, Phys. Rev. D 76 064004 (2007), arXiv:0705.1158[astro-ph]

$$f(R) = R + \frac{R^n(aR^n - b)}{1 + cR^n}$$

S. Nojiri and S.D. Odintsov, Phys. Rev. D 77 026007 (2008), arXiv:0710.1738[hep-th]

$$f(R) = R + \lambda R_0\left(\left(1 + \frac{R^2}{R_0^2}\right)^{-n} - 1\right)$$

A. Starobinsky, JETP Lett. 86 157 (2007), arXiv:0706.2041 [astro-ph]

$$f(R) = R - \lambda R_c \tanh\left(\frac{R}{R_c}\right)$$

S. Tsujikawa, Phys. Rev. D 77 023507 (2008), arXiv:0709.1391[astro-ph]

In some limit, GR is recovered, the local violations are avoided, and the non-linear part become important for large scales, also avoids matter instabilities...but this kind of models usually cross the phantom barrier, which may lead to future singularities.

Cosmological evolution in viable F(R)

Using the redshift as the independent variable:

$$1+z=\frac{a_0}{a(t)}$$

FLRW equations yield:

$$H^2(z)=\frac{1}{3f_R}\left[\kappa^2\rho_m(z)+\frac{R(z)f_R-f}{2}+3(1+z)H^2f_{RR}R'(z)\right]$$

$$(1+z)\rho_m'(z)-3(1+w_m)\rho_m(z)=0\ ,$$

where the last equation can be easily solved,

$$\rho(z)=\rho_0(1+z)^{3(1+w_m)}$$

Effective equation of state parameter:

$$w_{eff}=\frac{p_{F(R)}+p_m}{\rho_{F(R)}+\rho_m}=-1-\frac{2\dot{H}(t)}{3H^2(t)}=-1+\frac{2(1+z)H'(z)}{3H(z)}$$

Cosmological parameters:

$$\Omega_m=\frac{\rho_m}{\frac{3}{\kappa^2}H^2}\ ,\quad \Omega_{F(R)}=\frac{1}{3H^2}\left(\frac{RF_R-F}{2}-3H\dot{R}F_{RR}-3H^2F_R\right)$$

Cosmological evolution in viable F(R)

Hu-Sawicki model

$$f(R) = R - R_{HS}\frac{c_1(R/R_{HS})^n}{c_2(R/R_{HS})^n+1} \quad \text{where} \quad R_{HS} = \kappa^2\rho_m^0$$

$$\text{and} \quad n = 1\,, \quad c_1 = 2\,, \quad c_2 = 1$$

Initial conditions:

1. Assuming ΛCDM model at z=0,

$$H(z=0) = H_0 = 100\,h\ km\ s^{-1}\ Mpc^{-1}\ , h = 0.71 \pm 0.03$$

$$H'(z) = \frac{\kappa^2}{2}\frac{\rho_m}{H_0\ (1+z)h(z)}\ , \quad \rightarrow \quad H'(0) = \frac{\kappa^2}{2H_0}\rho_m^0 = \frac{3}{2}\Omega_m^0$$

2. Assuming a phantom expansion at z=0,

$$H(z=0) = H_0 = 100\,h\ km\ s^{-1}\ Mpc^{-1}\ , h = 0.71 \pm 0.03$$

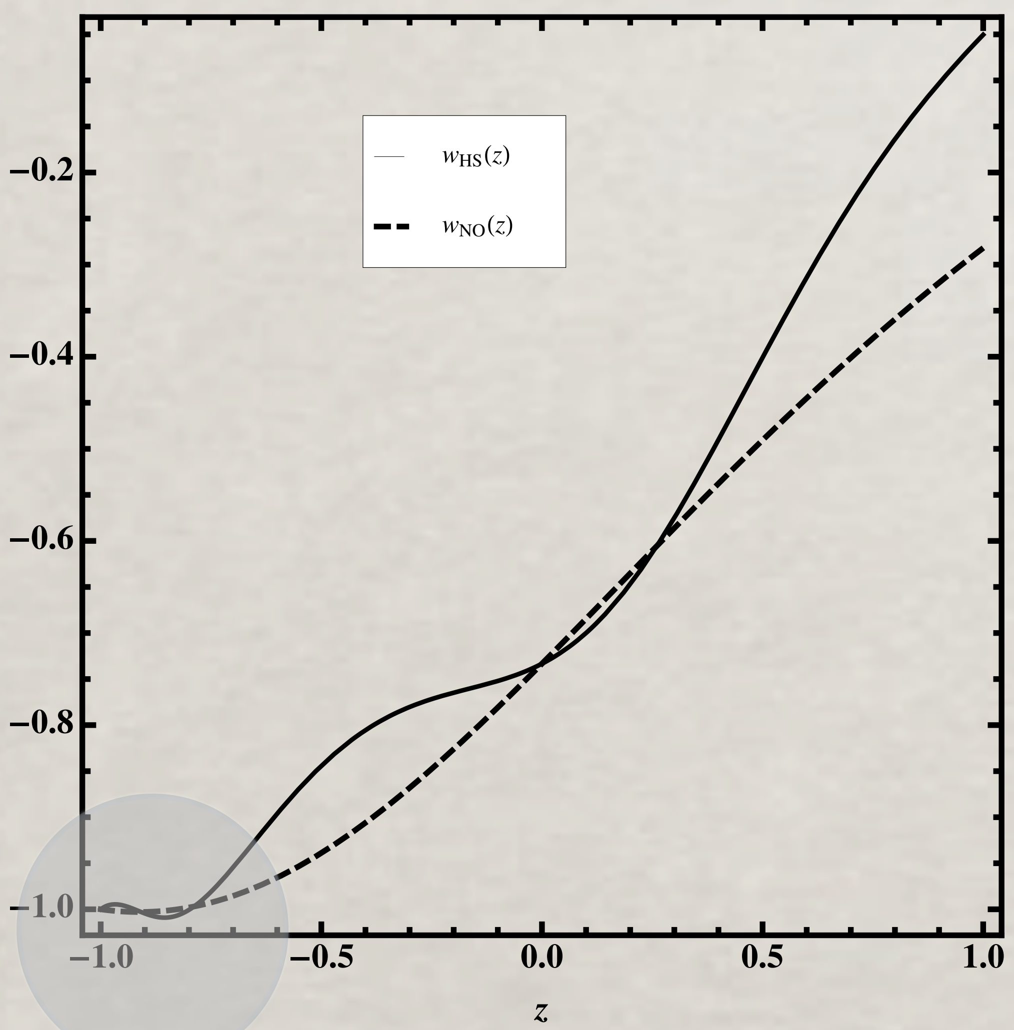
$$H'(0) \lesssim 0$$

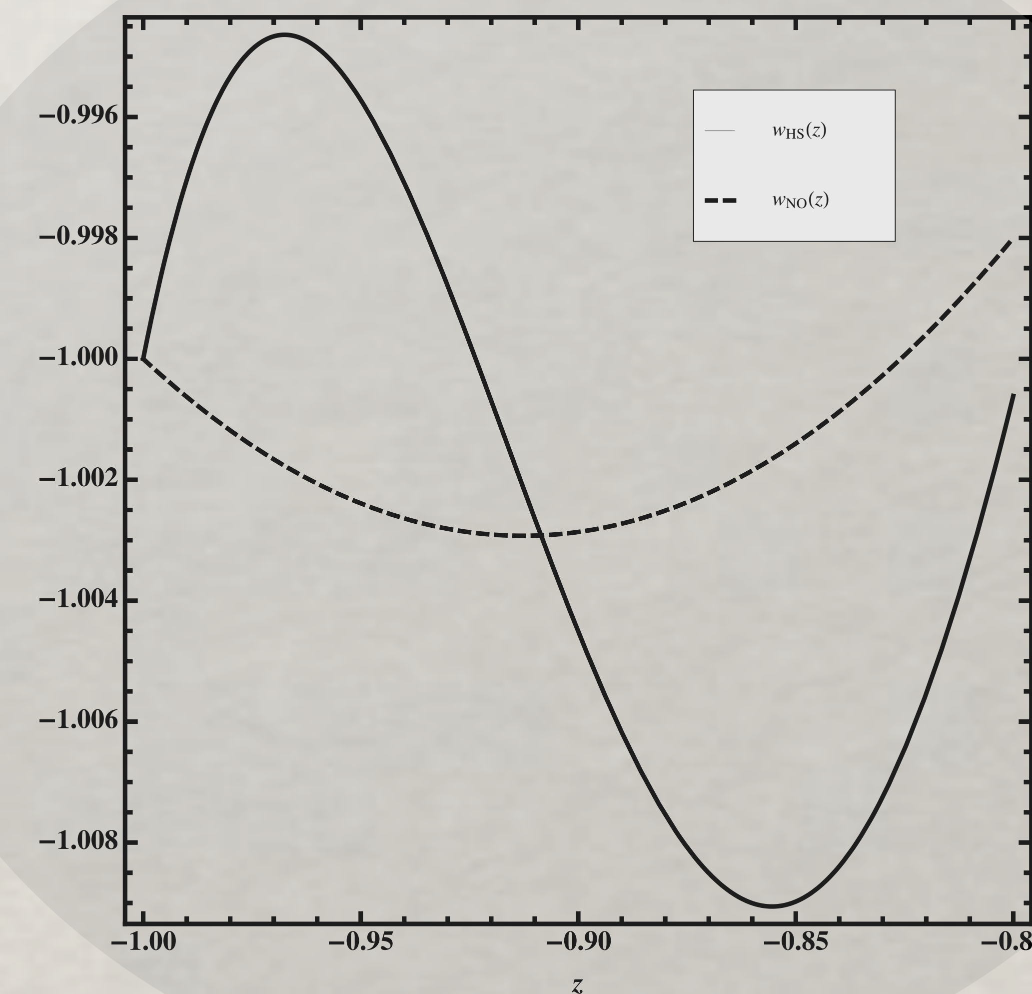
Cosmological evolution in viable f(R)

Hu-Sawicki and Nojiri-Odintsov models

1. ΛCDM initial conditions,

EoS parameter $w(z)$





The EoS parameter will cross the phantom barrier in the future.

K. Bamba, C. Q. Geng and C. C. Lee, JCAP 1011, 001 (2010)

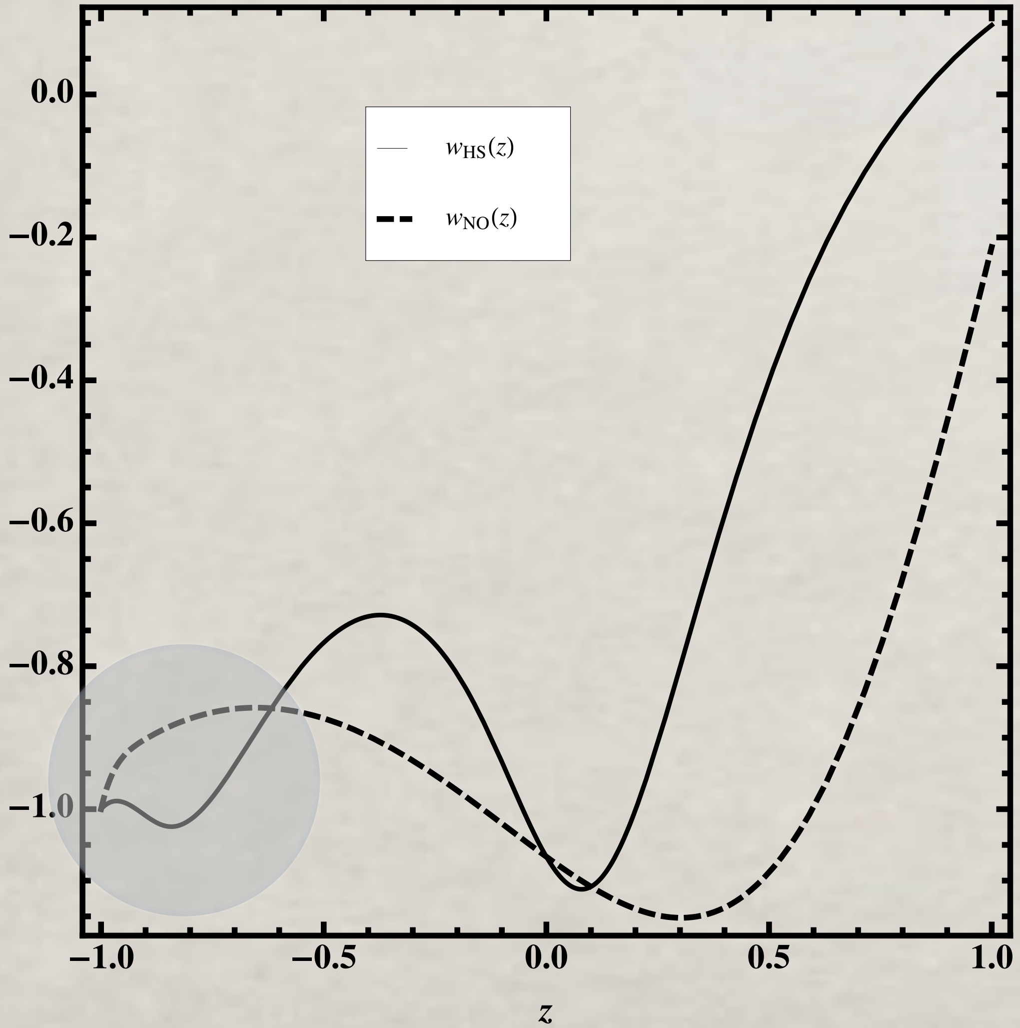
DSG, Class.Quant.Grav. 30 095008 (2013)

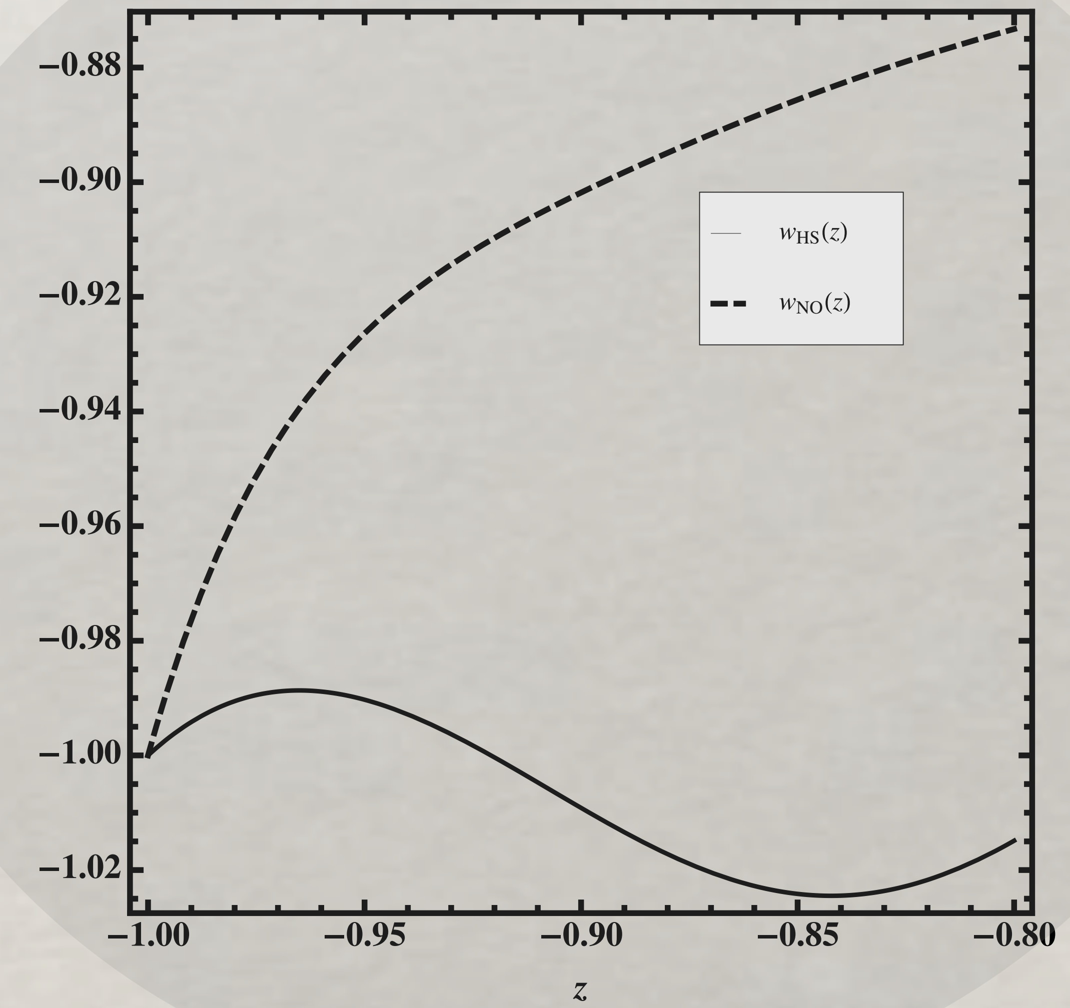
Cosmological evolution in viable f(R)

Hu-Sawicki and Nojiri-Odintsov model

2. Phantom initial conditions

EoS parameter



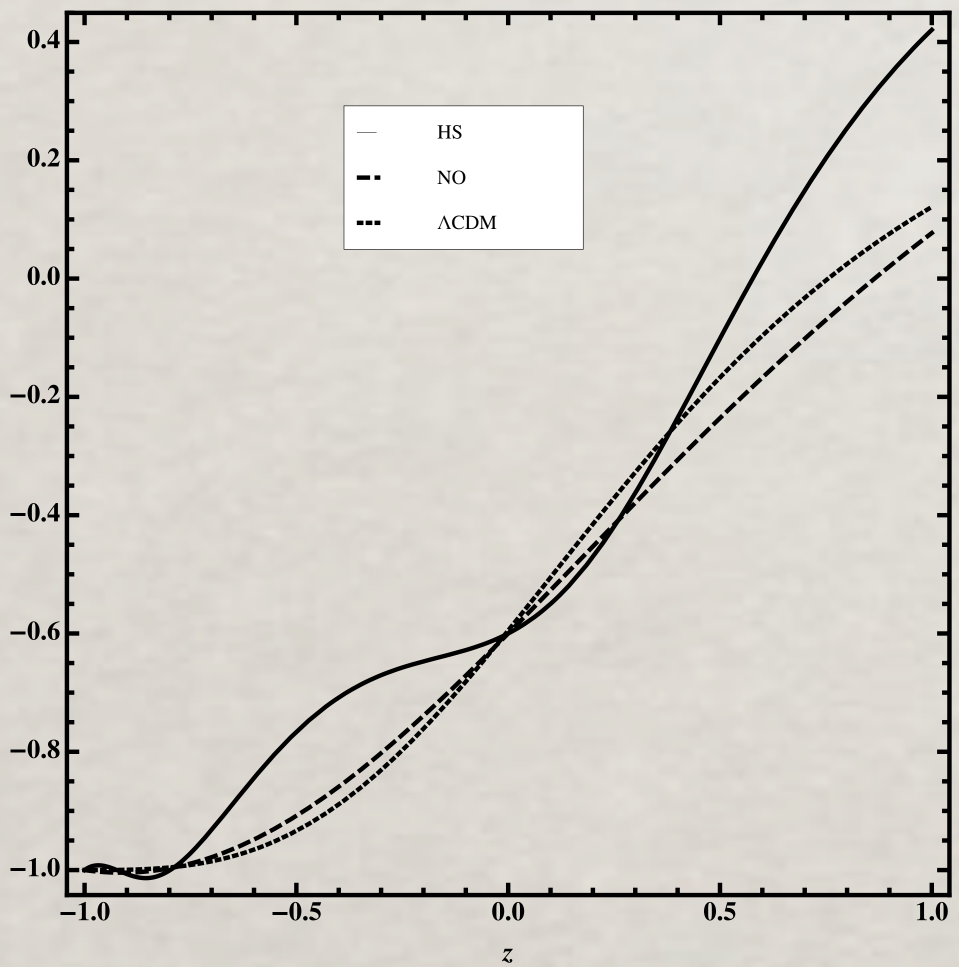


DSG, Class.Quant.Grav. 30 095008 (2013)

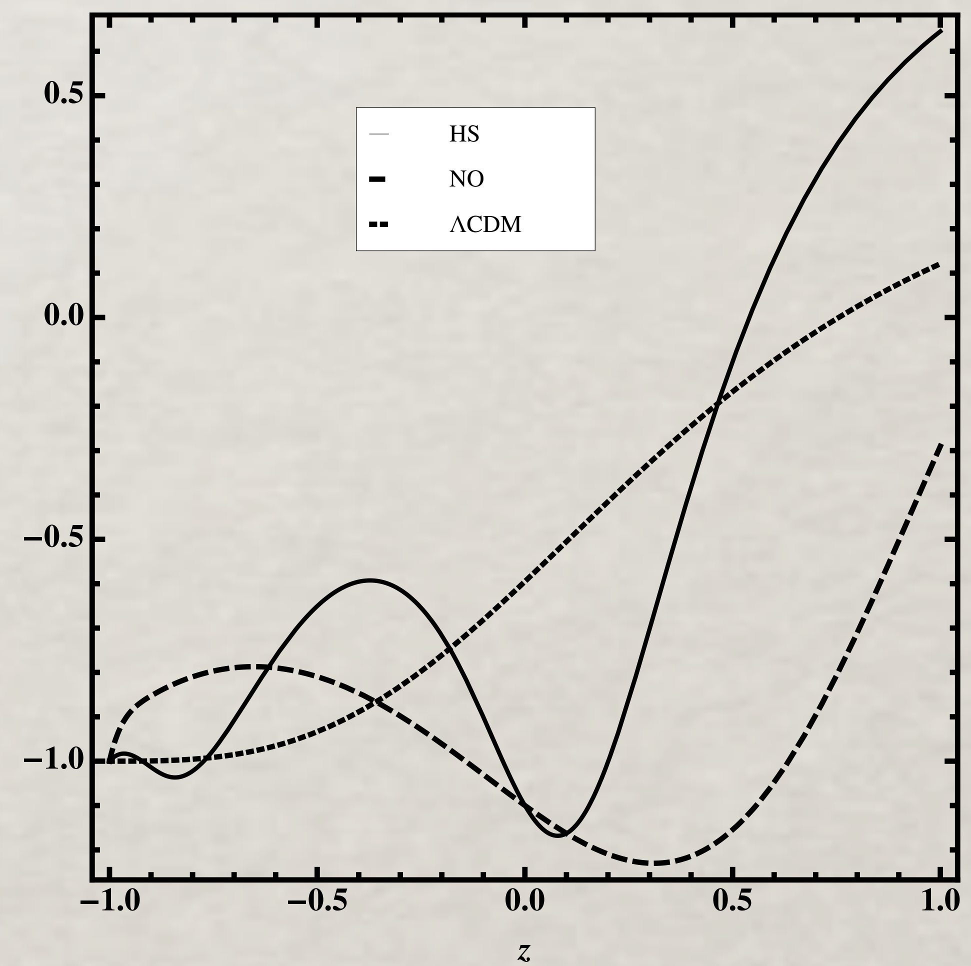
Cosmological evolution in viable f(R)

Hu-Sawicki and Nojiri-Odintsov model

Deceleration parameter $q = -a\ddot{a}/\dot{a}^2$



ΛCDM initial conditions



Phantom initial conditions

DSG, Class.Quant.Grav. 30 095008 (2013)

Scalar-tensor representation of f(R) gravity: presence of a sudden singularity

Action,

$$S = \int d^4x\sqrt{-g}\left[\phi\ R - V(\phi) + 2\kappa^2\mathcal{L}_m\right]$$

$$R = V'(\phi) \quad \rightarrow \quad \phi = \phi(R)\ ,\ \Rightarrow \quad f(R) = \phi(R)R - V(\phi(R))$$

FLRW equations,

$$3H^2 = \frac{1}{\phi}\left(\kappa^2\rho_m - 3H\dot{\phi} + \frac{1}{2}V(\phi)\right)\ ,$$

$$-3H^2 - 2\dot{H} = \frac{1}{\phi}\left(\kappa^2 p_m + \ddot{\phi} + 2H\dot{\phi} - \frac{1}{2}V(\phi)\right)\ .$$

Trace equation,

$$3\ddot{\phi} = \kappa^2(\rho_m - p_m) - \phi V' + 2V - 9H\dot{\phi}$$

In vacuum, the phase space can be described by,

$$\dot{H} = -2H^2 + \frac{1}{6}V'(\phi)\ , \quad \dot{\phi} = \frac{1}{3H}\left[-3H^2\phi + \frac{1}{2}V(\phi)\right]$$

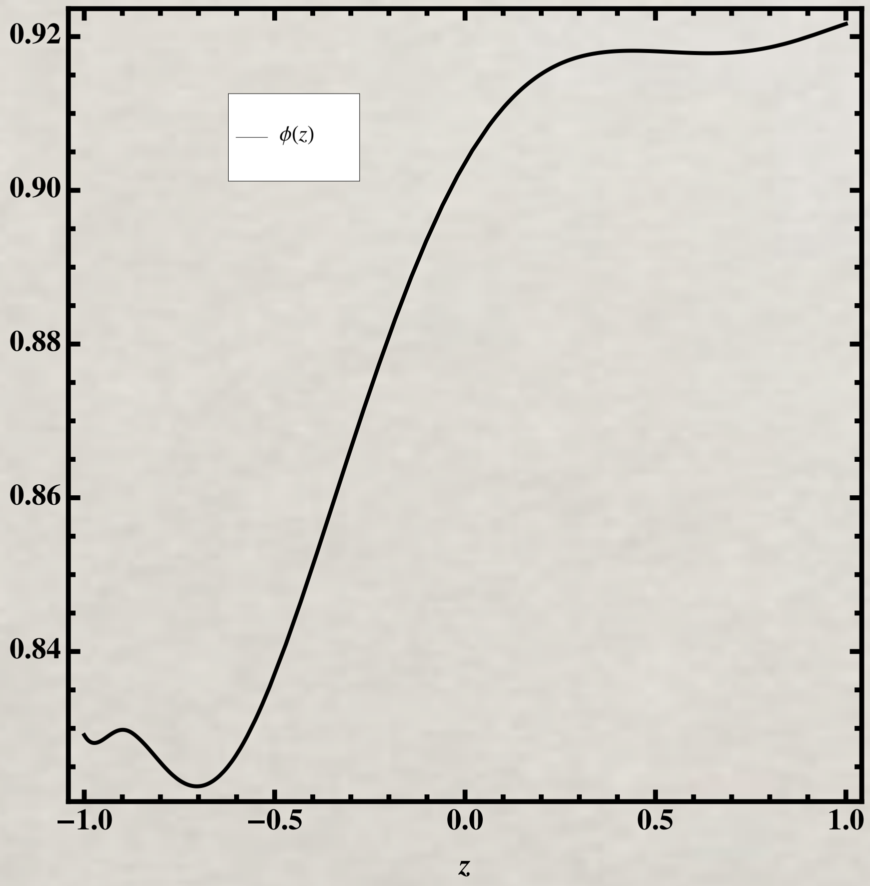
DSG, Class.Quant.Grav. 30 095008 (2013)

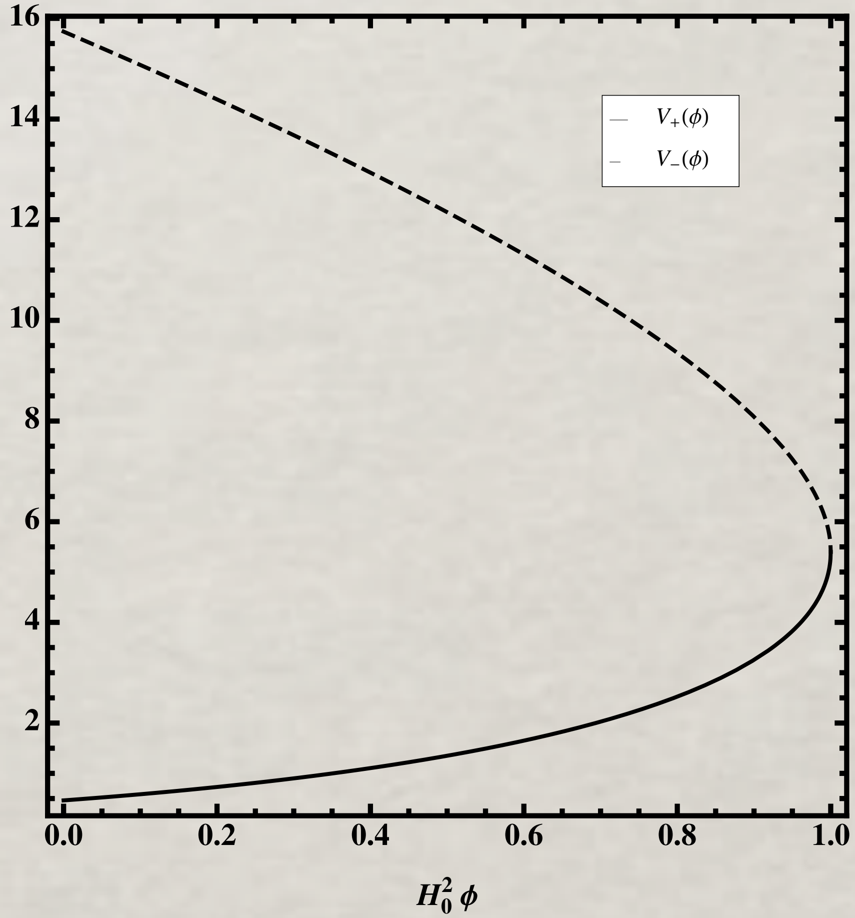
Scalar-tensor representation of f(R) gravity

Hu-Sawicki model

$$\phi_{HS} = 1 + \frac{c_1 c_2 R}{R_{HS}\left(1+\frac{c_2 R}{R_{HS}}\right)^2} + \frac{c_1}{1+\frac{c_2 R}{R_{HS}}}$$

$$V_{HS}(\phi) = \frac{1 + c_1 - \phi \pm 2\, c_2 \sqrt{c_1\,(1-\phi)}}{c_2} R_{HS}$$





S. A. Appleby, R. A. Battye and A. A. Starobinsky, JCAP 1006, 005 (2010), arXiv:0909.1737

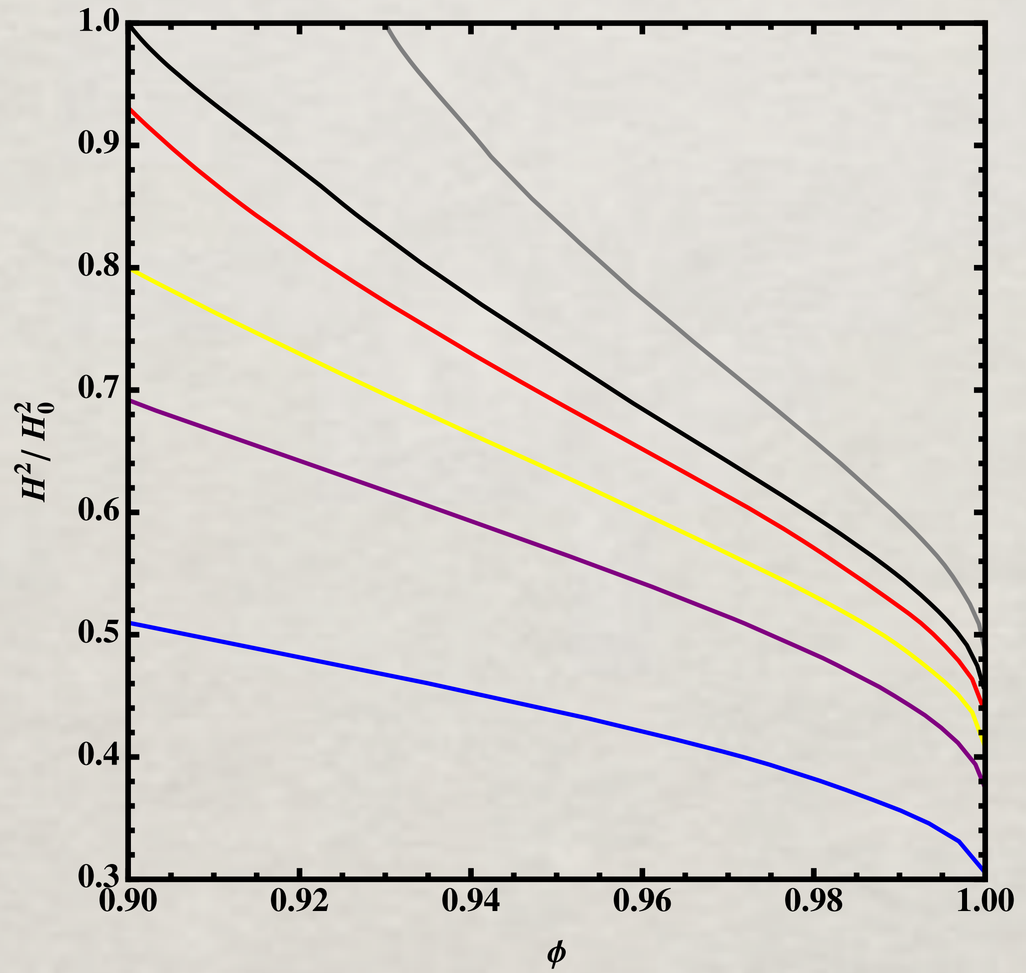
DSG, Class.Quant.Grav. 30 095008 (2013)

Scalar-tensor representation of f(R) gravity

Hu-Sawicki model

Phase space

$V_+(\phi)$



This branch of the potential makes the scalar field to tend to the boundary, where a sudden singularity occurs.

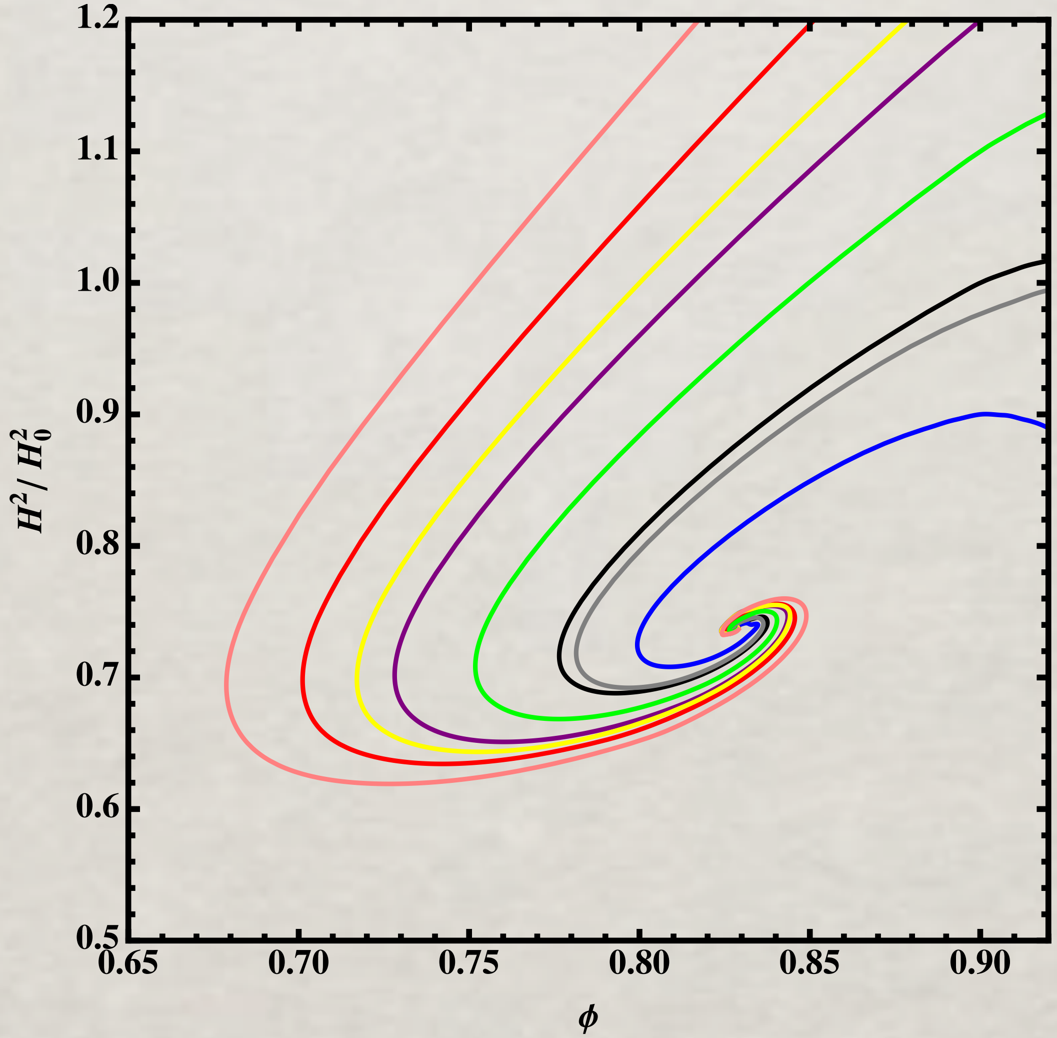
DSG, Class.Quant.Grav. 30 095008 (2013)

Scalar-tensor representation of f(R) gravity

Hu-Sawicki model

Phase space

$V_{-}(\phi)$



There is a an asymptotic focus, which corresponds to a stable de Sitter solution.

DSG, Class.Quant.Grav. 30 095008 (2013)

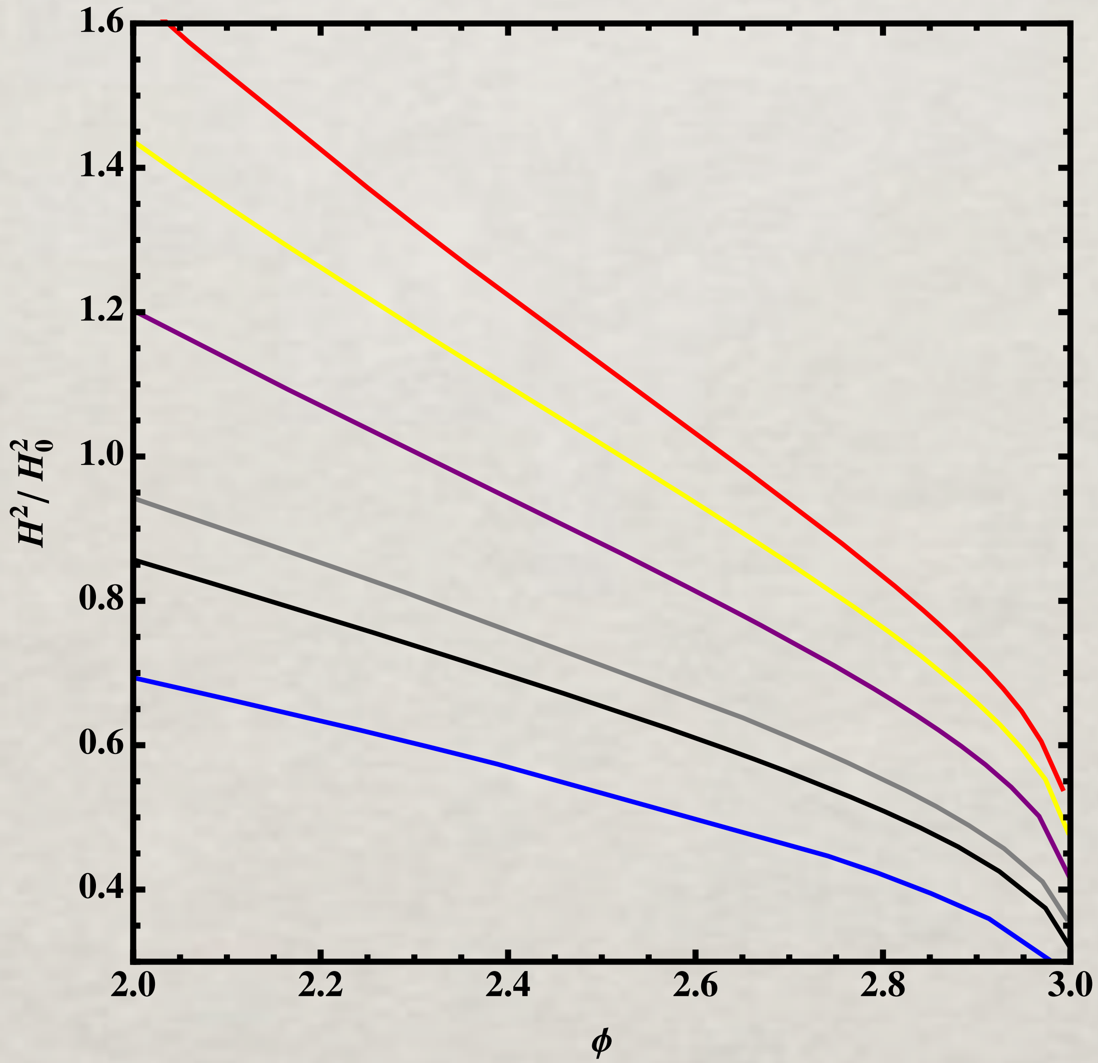
G. Cognola, E. Elizalde, S. D. Odintsov, P. Tretyakov, and S. Zerbini, Phys. Rev. D 79 044001 (2009)

Scalar-tensor representation of f(R) gravity

Nojiri-Odintsov model

Phase space

$V_+(\phi)$



The scalar field reaches its boundary, where a sudden singularity occurs.

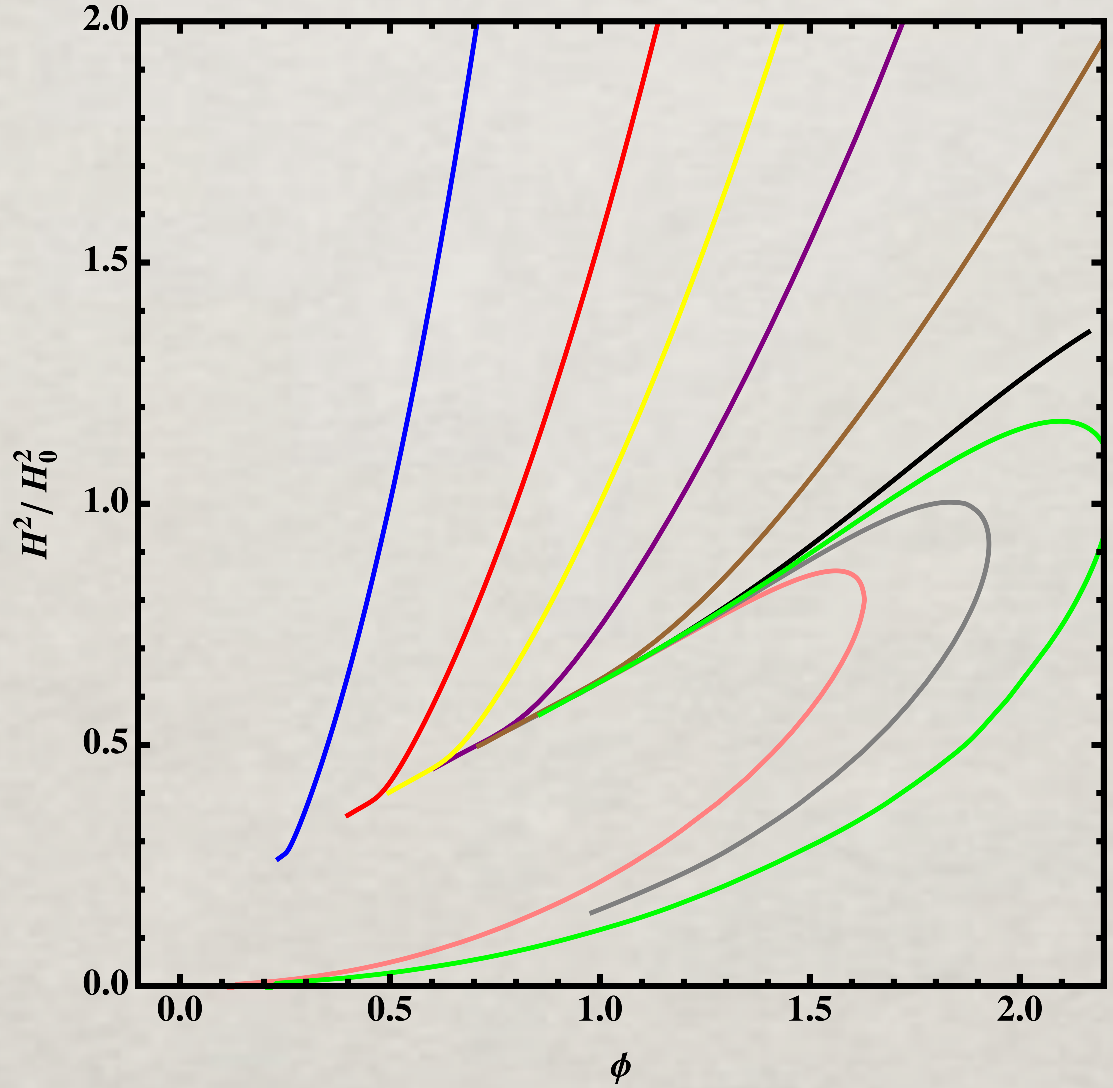
DSG, Class.Quant.Grav. 30 095008 (2013)

Scalar-tensor representation of f(R) gravity

Nojiri-Odintsov model

Phase space

$V_{-}(\phi)$



The Hubble parameter tends to a an asymptotically stable dS point.

DSG, Class.Quant.Grav. 30 095008 (2013)

The Little Rip

As the universe expands, the relative acceleration between two points separated by a comoving distance l is affected by the acceleration of the expansion. An observer located at a comoving distance l away from a mass m will measure an inertial force on the mass of

$$F_{\text{iner}} = ml\ddot{a}/a = ml\left(\dot{H} + H^2\right)$$

If the two particles are bounded by a constant force, when

$$F_{iner} > F_0 \longrightarrow$$

The binding system is broken and a "Rip"of the system occurs.

This usually occurs for singularities of the type of Big Rip (H diverges) and Type II singularities (H' diverges).

However, a break of the bounded system may occur with no future singularity, which is called the *Little Rip*.

It could occur even in cyclic cosmologies, where the Hubble parameter presents a bound, but whose strength during the accelerating phase may be strong enough to make a Little Rip to occur.

P. Frampton, K. Ludwick, R. Scherrer, Phys. Rev. D **84**: 063003,2011, arXiv:1106.4996

The Little Rip

Little Rip in *f(R)* gravity

Hubble parameter

$$H(t) = h_0 \mathrm{e}^{\alpha t} + h_1 \,, \quad \rightarrow \quad a(t) = a_0 \mathrm{e}^{4\beta \mathrm{e}^{\alpha t} + 6\alpha t}$$

Action

$$P(\phi) = \mathrm{e}^{4\beta \mathrm{e}^{\alpha\phi}} \,, \quad Q(\phi) = -6\alpha^2 (3 + 4\beta \mathrm{e}^{\alpha\phi})(3 + 8\beta \mathrm{e}^{\alpha\phi}) \mathrm{e}^{4\beta \mathrm{e}^{\alpha\phi}} \longrightarrow F(R) = \left[C_1 + C_2 \sqrt{4\frac{R}{R_0} + 75} \right] \mathrm{e}^{\sqrt{\frac{R}{12R_0} + \frac{25}{16}}}$$

One can set the time of the little rip dissolution occurrence when the gravitational coupling of the Sun-Earth system is broken due to the cosmological expansion by assuming a density for the f(R) terms adjusted with the dark energy density nowadays,

$$\rho_{F(R)} = \rho_0 \mathrm{e}^{2\alpha t} \qquad \text{where} \qquad \rho_{F(R)}(t_0) = \frac{3}{\kappa^2} H_0^2 \sim 10^{-47}\,\mathrm{GeV}^4$$

Time for the Little Rip

$$t_{\mathrm{LR}} = 13.73\,\mathrm{Gyrs} + \frac{29.93}{\alpha}$$

S. Nojiri, S. Odintsov and DSG, AIP Conf. Proc. **1458**, 207 (2011), arXiv:1108.0767.
A. López-Revelles, R. Myrzakulov and DSG, Phys. Rev. D **85**, 103521 (2012), arXiv:1201.5647.

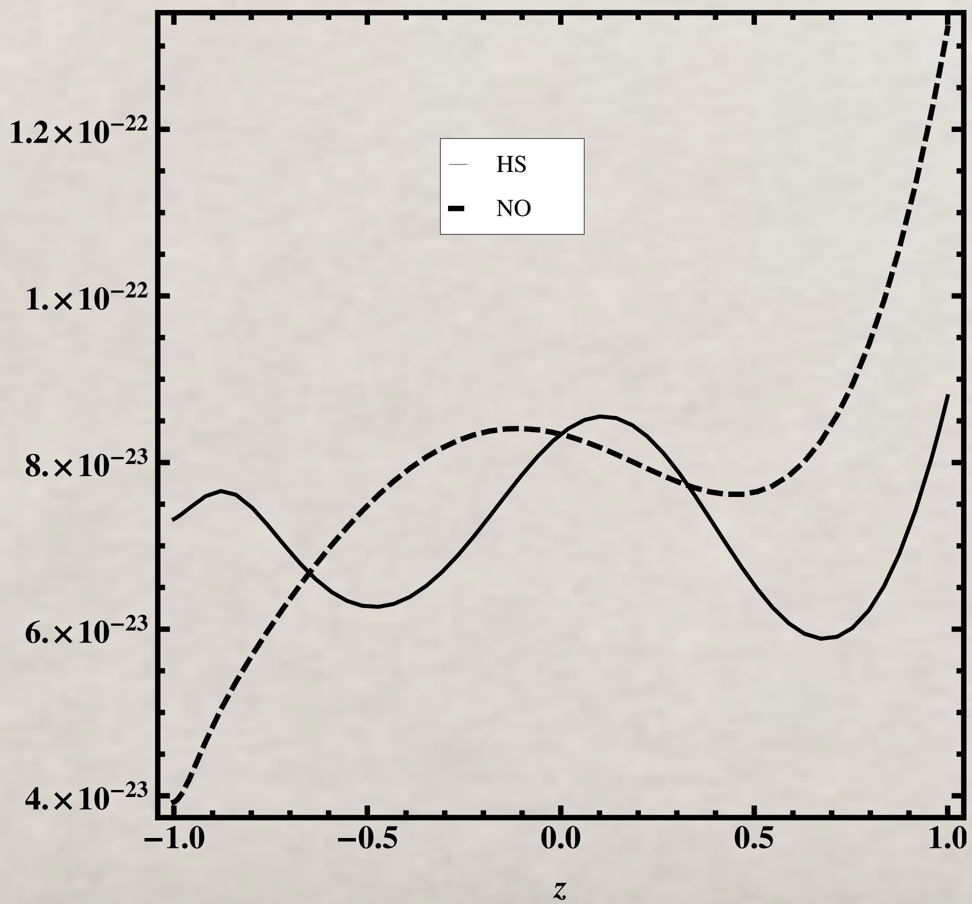
The Little Rip

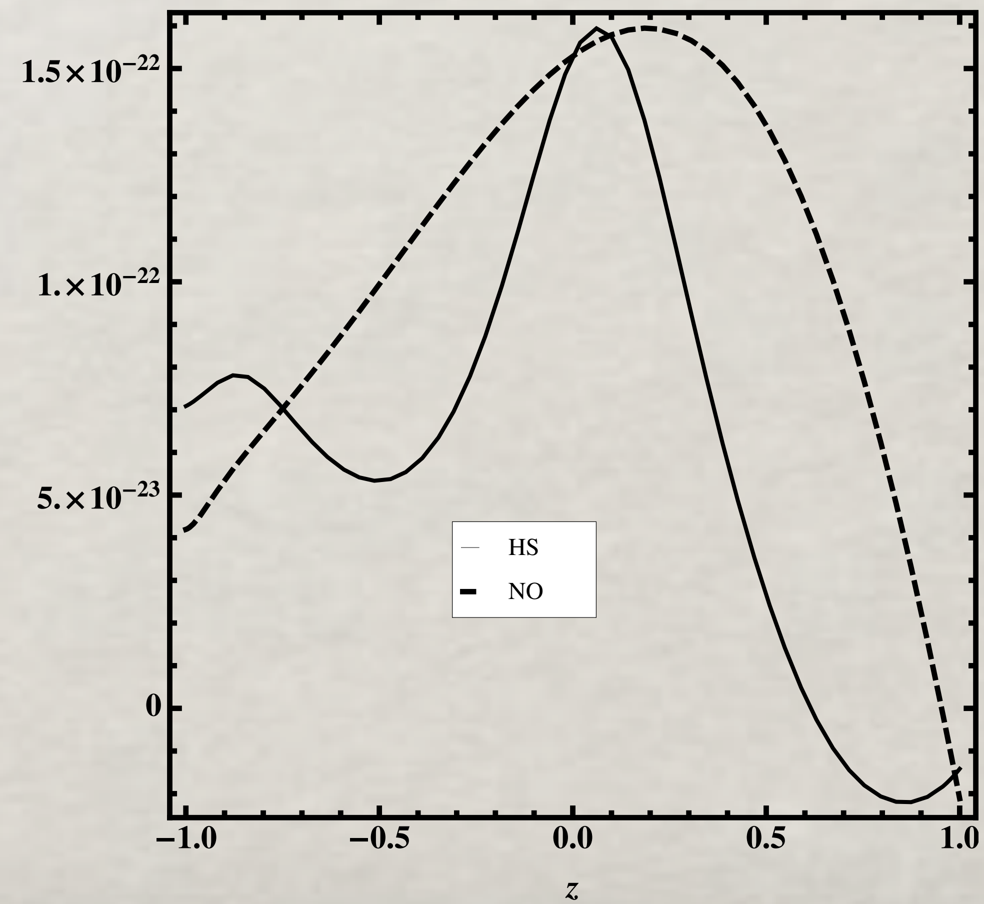
Little Rip in viable *f(R)* gravity

$$\frac{F_{cosm}}{F_N} = \frac{r^3}{G\ M_\odot} H_0^2 \left[h(z)^2 - (1+z)h(z)\ h'(z)\right]$$

Earth-Sun system

At z=0 $\longrightarrow$ $\dfrac{F_{cosm}}{F_N} = 1.4 \times 10^{-22} \left[h(0)^2 - h(0)h'(0)\right]$





DSG, Class.Quant.Grav. 30 095008 (2013)

Testing F(R) gravity with SNe Ia data

A simple model

$$f(R) = a\left(\frac{R}{R_0}\right) + b\left(\frac{R}{R_0}\right)^n$$

Viability conditions

To ensure the occurrence of a matter dominated epoch at high redshifts: $0.75 < n < 1.347$

$$f_{RR} > 0 \to \begin{cases} b > 0 \text{ and } n > 1 \text{ or} \\ b < 0 \text{ and } n > 0.75 \end{cases}$$

$$a > -bn\left(\frac{R}{R_0}\right)^{n-1}$$

Fixed point: matter dominated epoch

$$a = \frac{n\left[2n(1+q_0+2\delta_1) - 3(1+\delta_1)\right]}{(n-1)\left[3+n(8n-13)\right](q_0-1)(1+\delta_2)}$$

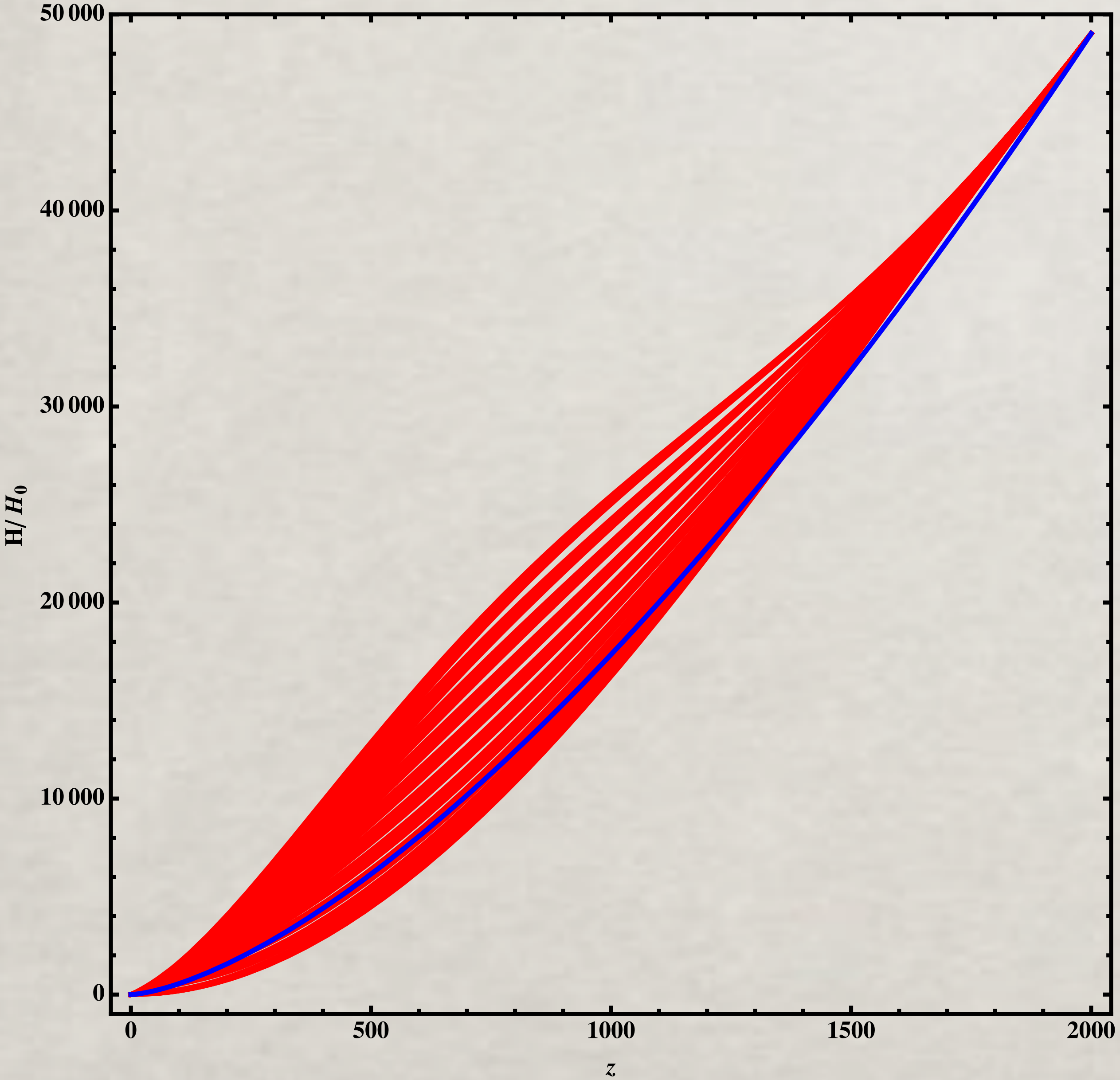
$$b = \frac{k(1-q_0)^{-n}\left[2n^2(q_0-1) - 3(1+\delta_1) + 4n(1+\delta_1)\right]}{(n-1)\left[3+n(8n-13)\right](1+\delta_2)}$$

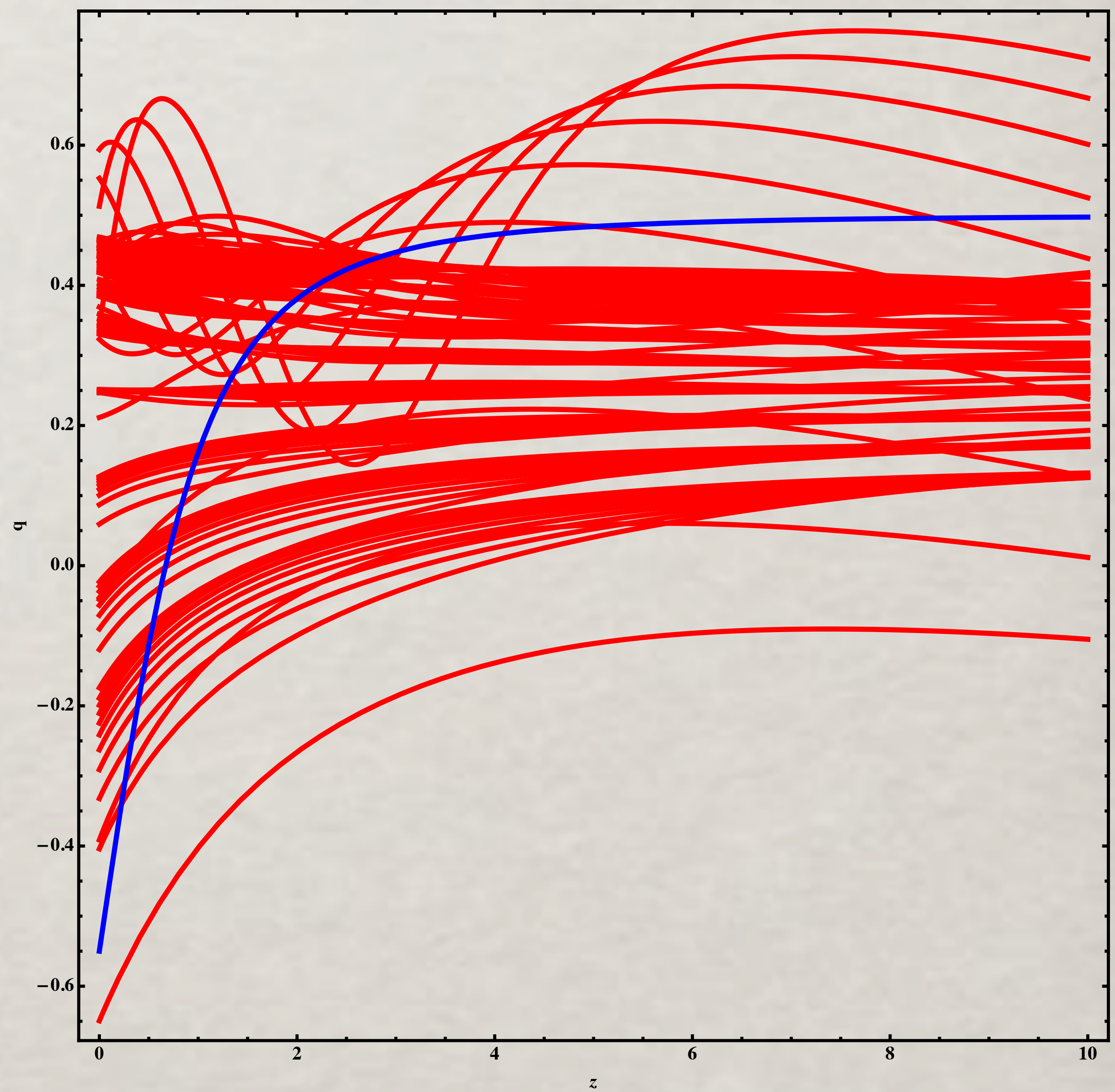
A. Cruz-Dombriz, P. Dunsby, DSG, in preparation

Testing F(R) gravity with SNe Ia data

A simple model

Fixing the initial conditions at $z = 2000$ to match the ΛCDM model

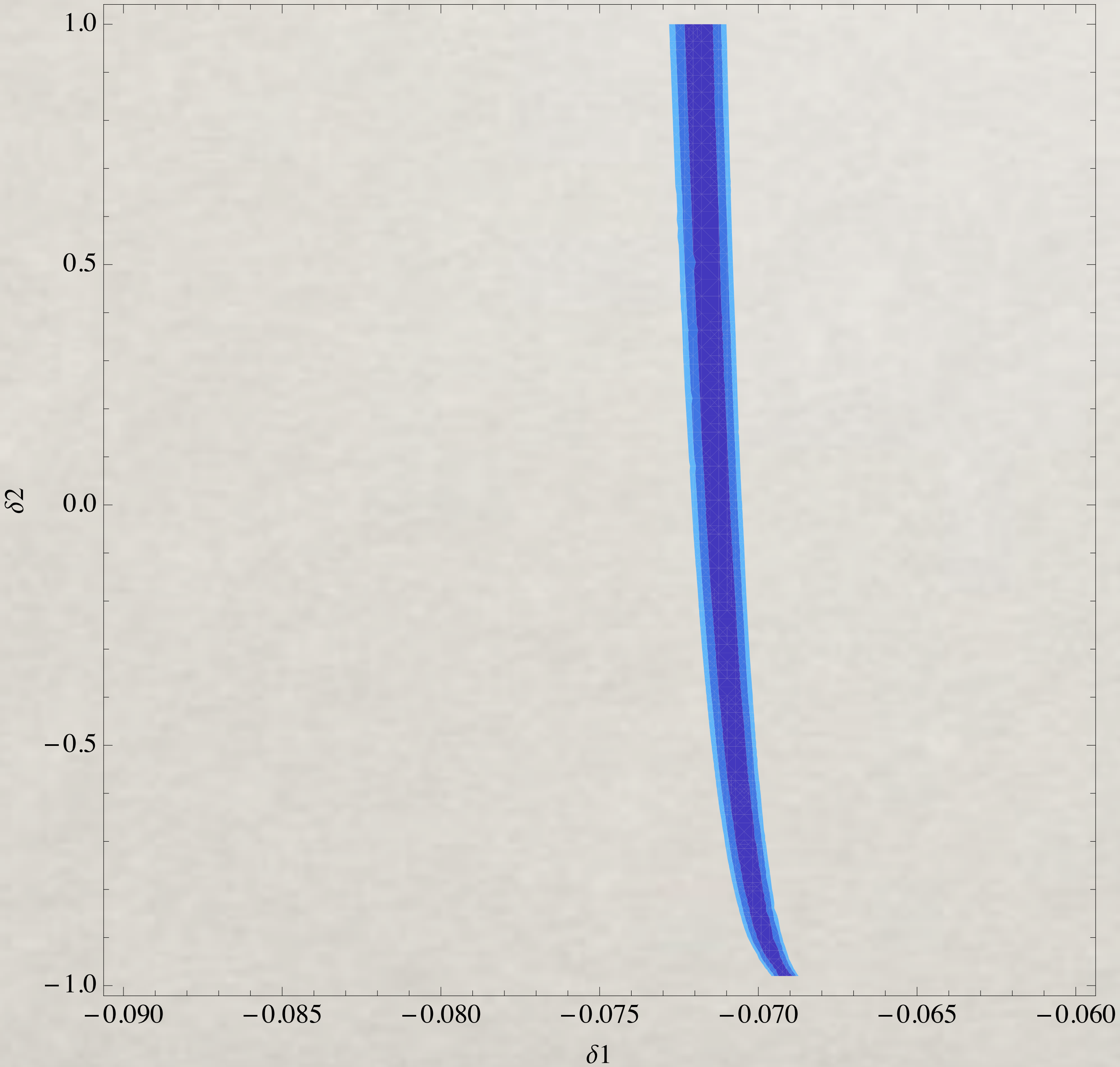




A. Cruz-Dombriz, P. Dunsby, DSG, in preparation

Testing F(R) gravity with SNe Ia data

Using the Union2 SNIa Dataset (557 Sne)



ΛCDM model,

$$\chi^2_{min} = 542.68 \ (\Omega_m = 0.27 \pm 0.02)$$

f(R) gravity,

$$\chi^2_{min} = 546.204 \ (\Omega_m = 0.3)$$

$$\delta_1 = -0.07186, \ \delta_2 = 1$$

A. Cruz-Dombriz, P. Dunsby, DSG, in preparation

F(R,T) GRAVITY

Action,

$$S = S_G + S_m = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\ \left(f(R,T) + \mathcal{L}_m\right)$$

Field equations,

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} - \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)f_R(R,T) = -\left(\kappa^2 + f_T(R,T)\right)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu}$$

where the energy-momentum tensor is defined as:

$$T^{(m)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}$$

$$T = T^{\mu}{}_{\mu}$$

Divergence of the field equations

$$\nabla^\mu T_{\mu\nu} = \frac{f_T}{\kappa^2 + f_T}\left[\frac{1}{2}g_{\mu\nu}\nabla^\mu T - (T_{\mu\nu} + \Theta_{\mu\nu})\nabla^\mu \ln f_T - \nabla^\mu\Theta_{\mu\nu}\right]$$

In general, the divergence of the equations is not null, which may give rise to an anomalous behavior of the matter content.

T. Harko, F. S. N. Lobo, S. Nojiri and S. D. Odintsov, Phys. Rev. D84, 024020 (2011)

F(R,T) GRAVITY

A particular case,

$$f(R,T) \;=\; f_1(R) + f_2(T)$$

Friedmann equations,

$$\mathrm{d}s^2 \;=\; a^2(\eta)\left(\mathrm{d}\eta^2 - \mathrm{d}\mathbf{x}^2\right)$$

$$-3\mathcal{H}f^{'}_{1R_0} + 3\mathcal{H}' f_{1R_0} - \frac{a^2}{2}f_{10} \;=\; -\kappa^2 a^2 \rho_0 + (1+c_s^2)\rho_0 a^2 f_{2T_0} + \frac{a^2}{2}f_{20}$$

$$f^{''}_{1R_0} + \mathcal{H}f^{'}_{1R_0} - (\mathcal{H}' + 2\mathcal{H}^2)f_{1R_0} + \frac{a^2}{2}f_{10} \;=\; -\kappa^2 a^2 c_s^2 \rho_0 - \frac{a^2}{2}f_{20} \;\; .$$

Continuity equation,

$$\rho^{'}_0 + 3\mathcal{H}\rho_0(1+c_s^2) = \frac{1}{\kappa^2 - f_{2T_0}}\left[(1+c_s^2)\rho_0 f'_{2T_0} + c_s^2\rho'_0 f_{2T_0} + \frac{1}{2}f'_{20}\right]$$

The usual continuity equation is recovered when,

$$f_2(T) = \alpha\sqrt{T} + \beta$$

F. Alvarenga, A. Cruz-Dombriz, S. Houndjo, M. Rodrigues, DSG, to appear in PRD, ArXiv:1302.1866

F(R,T) GRAVITY

Cosmological perturbations

$$\mathrm{d}s^2 = a^2(\eta)\left[(1+2\Phi)\mathrm{d}\eta^2 - (1-2\Psi)\mathrm{d}\mathbf{x}^2\right]$$

$$\hat{\delta}T^0_{\ 0} = \hat{\delta}\rho = \rho_0\delta\,, \quad \hat{\delta}T^i_{\ j} = -\hat{\delta}p\ \delta^i_{\ j} = -c_s^2\rho_0\delta^i_{\ j}\delta\,, \quad \hat{\delta}T^0_{\ i} = -\hat{\delta}T^i_{\ 0} = -\left(1+c_s^2\right)\rho_0\partial_i v$$

The perturbed equation can be reduced to,

$$\delta'' + \mathcal{H}\left[1 - \frac{3f_{2T_0}}{2\left(\kappa^2 - f_{2T_0}\right)}\right]\delta' + k^2\frac{f_{2T_0}}{2(\kappa^2 - f_{2T_0})}\delta + k^2\Phi - 3\Psi'' - 3\mathcal{H}\left[1 - \frac{3f_{2T_0}}{2\left(\kappa^2 - f_{2T_0}\right)}\right]\Psi' = 0$$

In the sub-Hubble limit, $k \gg \mathcal{H}$

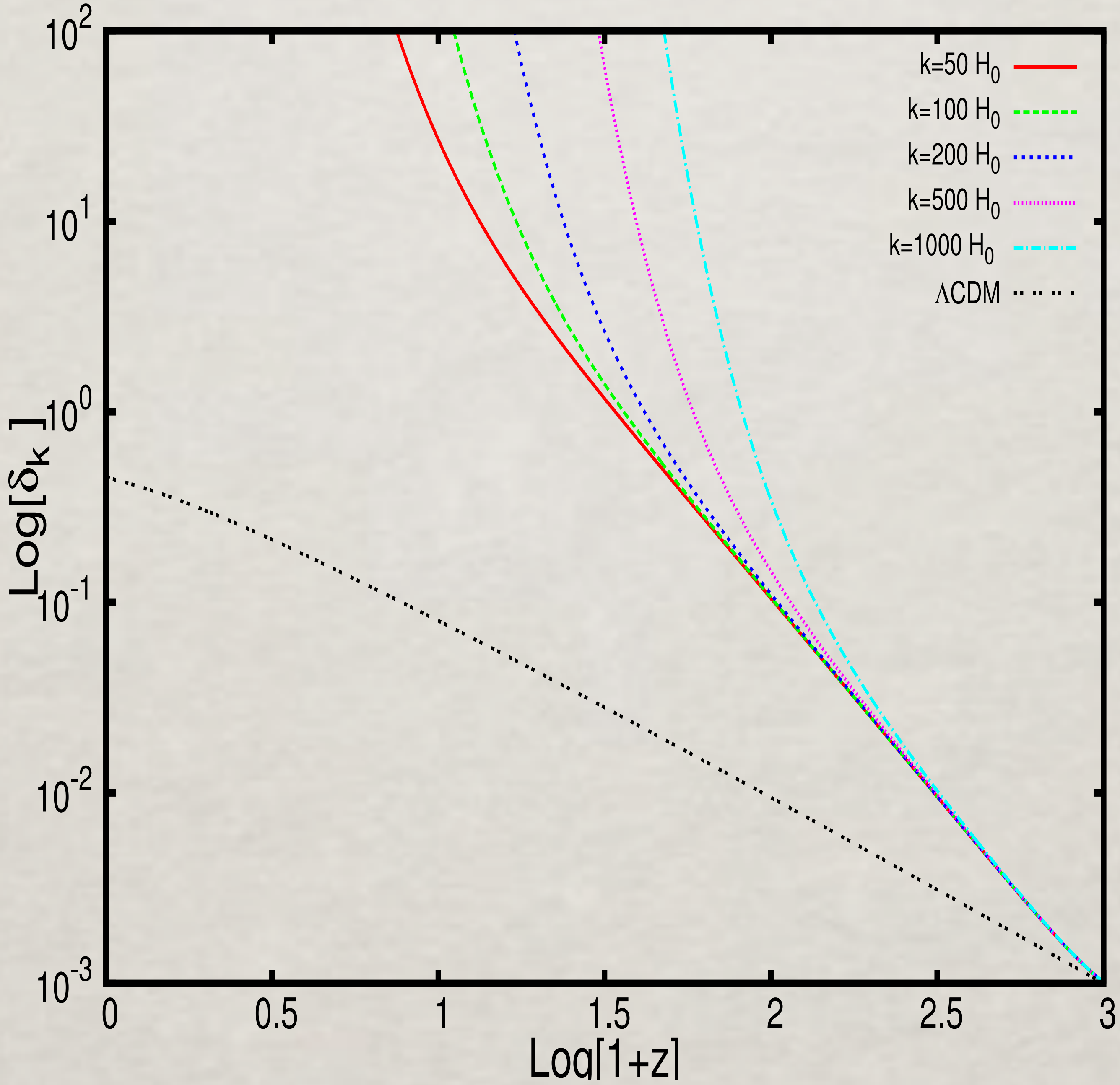
$$\delta'' + \mathcal{H}\left[1 - \frac{3f_{2T_0}}{2\left(\kappa^2 - f_{2T_0}\right)}\right]\delta' + \frac{1}{2}\left[k^2\frac{f_{2T_0}}{\left(\kappa^2 - f_{2T_0}\right)} - \left(\kappa^2 - f_{2T_0}\right)\frac{a^2\rho_0}{f_{1R_0}}\left(\frac{1 + 4\frac{k^2}{a^2}\frac{f_{1R_0R_0}}{f_{1R_0}}}{1 + 3\frac{k^2}{a^2}\frac{f_{1R_0R_0}}{f_{1R_0}}}\right)\right]\delta = 0$$

F. Alvarenga, A. Cruz-Dombriz, S. Houndjo, M. Rodrigues, DSG, to appear in PRD, ArXiv:1302.1866

F(R,T) GRAVITY

Evolution of the matter perturbations in the quasi-static limit

$f_A(R_0, T_0) = R_0 + \alpha T_0^{1/2}$



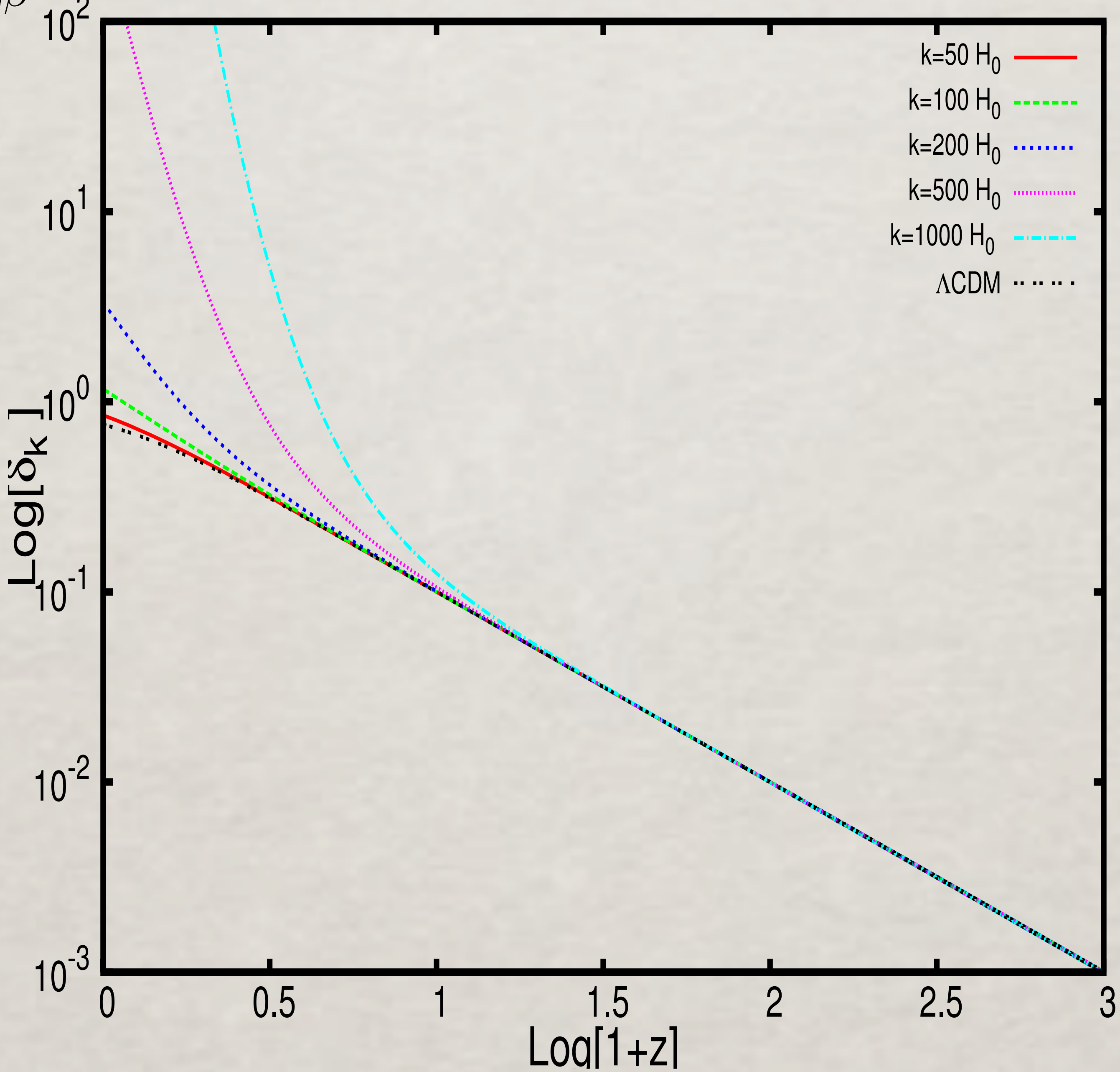
F. Alvarenga, A. Cruz-Dombriz, S. Houndjo, M. Rodrigues, DSG, to appear in PRD, ArXiv:1302.1866

F(R,T) GRAVITY

Evolution of the matter perturbations in the quasi-static limit

$$f_B(R_0,T_0) = R_0 + \alpha\, T_0^{1/2} - 2\beta$$

$\alpha < 0$



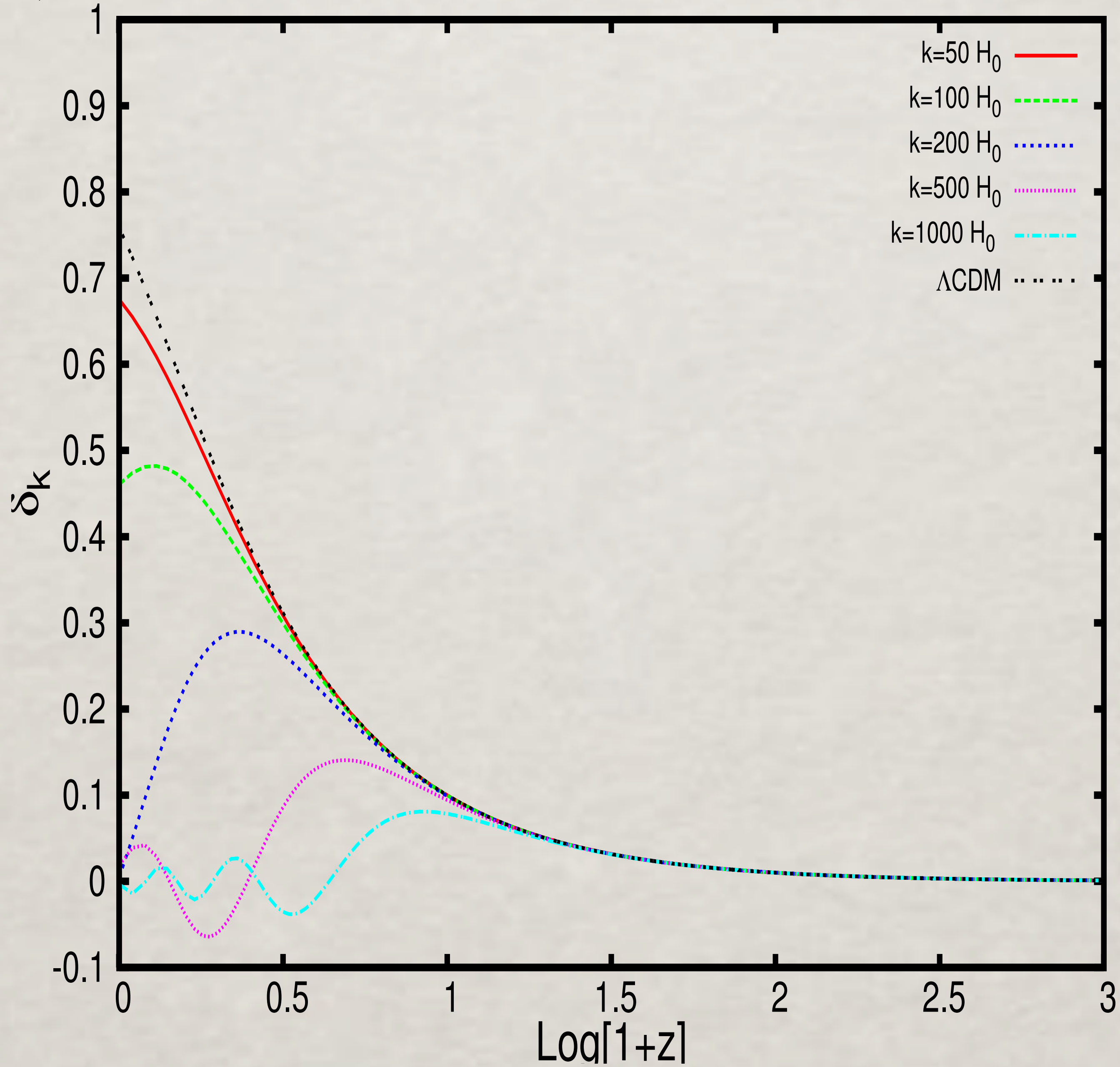
F. Alvarenga, A. Cruz-Dombriz, S. Houndjo, M. Rodrigues, DSG, to appear in PRD, ArXiv:1302.1866

F(R,T) GRAVITY

Evolution of the matter perturbations in the quasi-static limit

$f_B(R_0, T_0) = R_0 + \alpha T_0^{1/2} - 2\beta$

$\alpha > 0$



F. Alvarenga, A. Cruz-Dombriz, S. Houndjo, M. Rodrigues, DSG, to appear in PRD, ArXiv:1302.1866

Summary

- Viable modified gravities reproduce a sudden singularity, which may be circumvented in the scalar-tensor picture.

- The transition to a phantom phase probably occurs in the class of the so-called viable f(R) gravities, where the EoS parameter presents an oscillating behavior, but no future singularity or *Little Rip* occurs.

- The analysis of a simple model of f(R) gravity, and its test with the Sne Ia reveals that the f(R) gravities can describe late-time acceleration with great accuracy.

- f(R,T) gravities seem to be ruled out, since the non-standard coupling among matter and gravity introduce an anomalous behavior in the matter perturbations.